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AMPLITUDE AND PHASE RESONANCE IN A PARALLEL CIRCUIT

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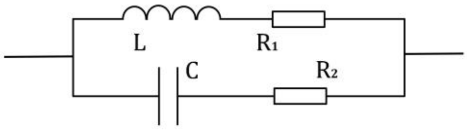
Let us consider a parallel oscillatory circuit in which there are active energy losses in each of the two branches – capacitive and inductive.



For such a circuit, two concepts of resonance can be considered - phase resonance and amplitude resonance, although most textbooks and reference books on electrical engineering and radio engineering are limited to considering only phase resonance. When a sinusoidal voltage is applied to such a circuit, the current passing through will be determined by the complex admittance $Y(\omega)$ of the circuit, which depends on the parameters of the circuit and the frequency of the input voltage. The phase resonance frequency is a frequency ω at which there is no phase shift between the voltage and current. This means that $\mathrm{Im}(Y(\omega)) = 0$. The amplitude resonance frequency is a frequency at which the amplitude of the current passing through the circuit will be minimal (current resonance) or maximal (voltage resonance). The main problem in studying the amplitude resonance of the circuit under consideration was the lack of exact formulas for finding the extreme points of admittance. In theses [1] such a formula for the amplitude resonance frequency of the circuit under consideration was given for the first time, and in article [2] its proof is given. Admittance $Y(\omega)$ can be written as

$$Y(\omega) = \frac{1}{R_1 + j\omega L} + \frac{j\omega C}{1 + j\omega C R_2} = \sqrt{\frac{C}{L}}\left(\frac{1}{a + jt} + \frac{jt}{1 + jbt}\right) =$$

$$= \sqrt{\frac{C}{L}}\left[\left(\frac{a}{a^2 + t^2} + \frac{bt^2}{1 + b^2 t^2}\right) + jt\left(\frac{1}{1 + b^2 t^2} - \frac{1}{a^2 + t^2}\right)\right]$$

$$t = \omega\sqrt{CL} = \frac{\omega}{\omega_0} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad a = R_1\sqrt{\frac{C}{L}} \qquad b = R_2\sqrt{\frac{C}{L}}$$

In such variables, the dependence of admittance on frequency is determined by only two parameters (a, b). In this case, the reduced frequencies of phase t_1 and amplitude t_2 resonance are calculated as

$$t_1^2 = \frac{1-a^2}{1-b^2} \qquad t_2^2 = \frac{\sqrt{1+2a^2+ab} - a^2\sqrt{1+2b^2+ab}}{\sqrt{1+2b^2+ab} - b^2\sqrt{1+2a^2+ab}}$$

From these formulas, we can determine the conditions for the existence of the corresponding resonance (regions on the plane of variables (a,b)). These regions are shown in Figure 1. The symmetric curves of degree six that bound region I intersect the axes at the points $\sqrt{1+\sqrt{2}} \approx 1.55377$

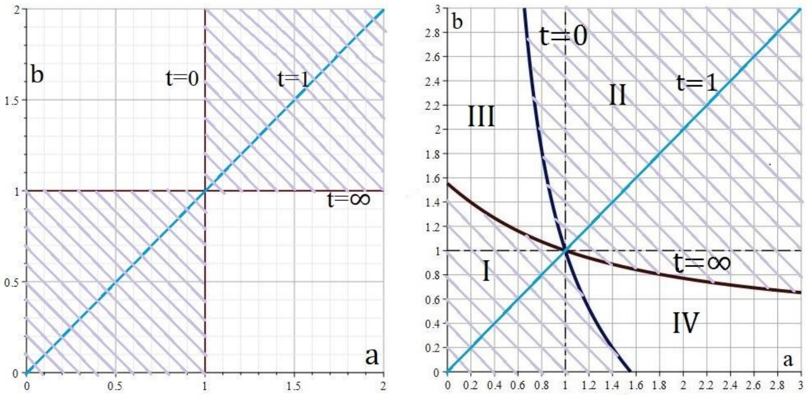


Figure 1. The region of existence of phase (left) and amplitude (right) resonance

From this figure, it is clear that there are parameter values (a,b) for which there is an amplitude resonance, but no phase resonance. In region I there is a current resonance (minimum conductivity), and in region II there is a voltage resonance (maximum conductivity). In regions III and IV there is neither amplitude nor phase resonance. Figure 2 shows typical graphs of the amplitude-frequency and phase-frequency characteristics corresponding to points (a,b) from different parameter regions.

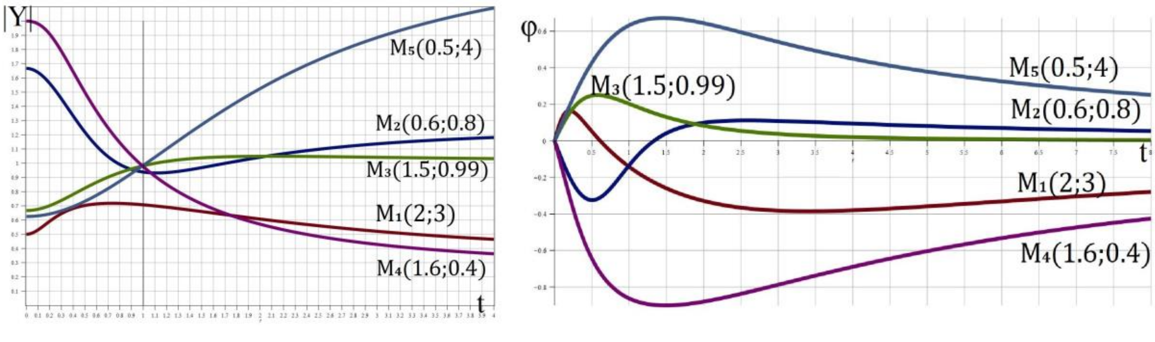


Figure 2 Typical graphs of amplitude-frequency (left) and phase-frequency (right) characteristics for different points of the plane (a,b)

References

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