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ASYMPTOTIC BEHAVIOR OF EXTREME VALUES IN *M* |*G*|1 SERVICE SYSTEM

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A single-channel mass service system (MSS) is considered, in which requests for service are received in moments $t_0 < t_1 < t_2 < \cdots < t_i < \cdots$. We denote by W_i the waiting time of the *i*-th request before the start of service. By the length of the queue we will understand the total number of demands that are being serviced or waiting for it. Let *Q*(*t*) denote the length of the queue at time *t*. The waiting time *Wi* and the queue length *Q*(*t*) are two main characteristics that are usually studied in mass service theory. The first problems about the properties of the quantities *Wi* and *Q*(*t*) appeared at the beginning of the 20th century. Their solutions are associated with the names of Erlang A., Pollaczek F., Khintchin A. (see [1], [2]). Let's put

$$\underline{Q}(t) = Q(s), \ \ \underline{W_n} = W_i \ .$$

It is clear that the study of extreme values of $\underline{W_n}$ and $\underline{Q}(t)$ is also of great interest for practical applications. Therefore, the values of W¯_n and Q¯(t) were studied in many works (see, for example, [1], [2], [3], [4], [5] and review [6]).. Usually, in such studies, the classical system *M*|*M* | 1 was considered. In this report, the mentioned results are transferred to a more general service system **M |G| 1.**

System *M* |*G*| **1** is a single-channel system, which receives a Poisson flow of requests with intensity λ = 1/*a*, and the service time has an arbitrary distribution $P(< x) = G(x)$.. Assume that E*η* = *b* < ∞ and next condition is fulfilled:

$$\rho = \frac{b}{a} < 1. \qquad (1)$$

Let $q(u) = P(\underline{Q}(T_1) \geq u)$– the probability of exceeding the level *u* by the process *Q*(*t*) in one regeneration cycle. We will assume that *u* runs through positive integers. To find the asymptotic of the quantity *q*(*u*) on the service time of the request *η*, we impose the following conditions:

$s_0,\ 0 < s_0 \le \infty$, such for $0 < s < s_0$ $\mathbf{E} \exp(s\eta) < \infty$, (2)

$$E\, exp\, exp\,(s) \uparrow \infty \text{ при } s \uparrow s_0 . \tag{3}$$

It is known [ ] that under conditions (2), (3) the following asymptotic formula holds:

$$q(u) = \left(C_1 + o(1)\right) exp\, exp\left(-\gamma_Q u\right), \tag{4}$$

where $\gamma_Q = ln\, ln\left(1 + \frac{\beta}{}\right),\ \beta > 0$ is the root of the equation

$$Eexp(\beta) = 1 + \frac{\beta}{}, \tag{5}$$

constant C_1 does not depend on u, $0 < C_1 < \infty$.

Theorem Let the condition (1) be fulfilled for the service system M|G|1, and the service time η satisfies the conditions (2), (3). Then

$$P\left(\frac{\gamma_Q \underline{Q}(t) - \alpha_m(t)}{L_{m+1}(t)} = 1\right) = 1, \tag{6}$$

$$P\left(inf_{\ t\to\infty}\left(\gamma_Q \underline{Q}(t) - \left(L_1\left(\frac{t}{M_T}\right) - L_3\left(\frac{t}{M_T}\right) + C\right)\right) = \kappa\right) = 1, \tag{7}$$

where γ_Q is defined in equations (4), (5), $C = \ln C_1$, κ is some non-random constant, $\kappa \in [-\gamma_Q, 0]$, $M_T = \frac{1}{} + \frac{b}{1-b} = \frac{1}{(1-\rho)}$.

$$L_0(t) = t, , \dots, L_m(t) = ln\, ln\, L_{m-1}(t), \dots, m \in \mathbb{N},\ \alpha_m(t) = \sum_{i=1}^{m} L_i(t),$$

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