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SOLVING THE RIDDLE OF M. C. ESHER'S LITHOGRAPHY REPTILES'

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A rule has been proposed for constructing a figured tile that stylizes images of plants and animals and fills the plane without overlaps or gaps with translations and rotations of its repetitions. The construction of a figured tile that generalizes the zoomorphic form in the lithograph by M. K. Escher 'Reptiles' is considered. The proposed rule was applied to compose an ornament stylizing M. C. Escher's lithograph 'Reptiles'.

Key words: *ornaments, figured tiles in the shape of animals and plants, stylization of prints by M. C. Escher.*

INTRODUCTION

The problem of covering a plane with figures of the same shape without overlaps and gaps came to us from ancient times. It is at least 5 thousand years old. Ornaments consisting of stylized figures of animals and plants that completely fill the plane were widespread in the decorative art of the Ancient East, in particular in the art of Ancient Egypt and Persia. Meanwhile, solving the problem of covering a plane with figured tiles of the same shape without overlaps and gaps has both theoretical and practical importance. Its theoretical importance lies in determining the conditions to which correspond to figured tiles that completely fill a plane, and its practical importance lies in the fact that knowledge of the laws of symmetry to which figured tiles is submitted allows one to discover new types of ornaments intended for textile production. Therefore, the discovery of the laws of symmetry, which correspond to a figured tile that fills a plane without overlaps and gaps with translations, rotations or reflections of its repetitions, is an urgent task for both geometers developing the theory of ornament and designers creating new fabric patterns for the textile industry.

PURPOSE

The purpose of the article is to classify ornaments according to crystallographic symmetry groups on the plane, discovered by the Russian crystallographer E. S. Fedorov, and to connect the symmetry groups of ornaments with the groups of plane movements that describe the construction of their repeating figures.

RESULTS AND DISCUSSION

In our opinion, solving the riddle of M. C. Escher's work or searching for an answer to the question: 'How did M. C. Escher create such famous prints as

‘Riders’, ‘Day and Night’, ‘Sky and Water’ or ‘Reptiles’?’, – we must begin by studying the symmetry groups of the ornaments he created [1–3].

Let's apply proposed rule for composing a zoomorphic form that fills a plane without overlaps and gaps with translations and rotations of its repetitions, to the construction of an ornament stylizing M. C. Escher’s lithograph ‘Reptiles’.

Let's show in Fig. 1, a first version of the ornament stylizing M. C. Escher’s lithograph ‘Reptiles’. It is remarkable that the ornament is a figure that inscribes into a regular hexagon. Therefore, it can be considered as a consequence of tiling the plane with groups of zoomorphic forms, carried out by translations in six directions, defined by perpendiculars to the sides of a regular hexagon.

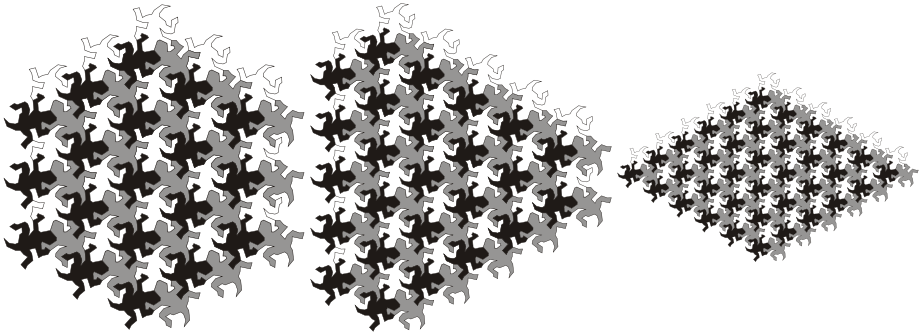
It is obvious that the ornament shown in Fig. 1, a does not have a single plane of symmetry, that is, it does not have a single reflection symmetry group. At the same time, it has rotational symmetry with 3rd order symmetry axes. This means that the ornament we are considering can be superposed with itself by rotating it around the axis of symmetry at an angle of 120°. Additionally, the ornament shown in Fig. 1, a, has translation symmetry with six translation axes defined by perpendiculars to the sides of a regular hexagon. Consequently, it can be superposed with itself using translation in one of the six directions specified by the translation axes.

Let's show in Fig. 1, b is the second variant of the ornament, stylizing M. C. Escher’s lithograph ‘Reptiles’ and representing a figure that inscribes into a regular triangle. Moreover, the figure is formed by tiling the plane with groups of zoomorphic forms, carried out by translations in three directions, specified by perpendiculars to the sides of a regular triangle.

It is obvious that the ornament shown in Fig. 1, b does not have a single plane of symmetry, that is, it does not have a single reflection symmetry group. At the same time, it has rotational symmetry with 3rd order symmetry axes. This means that the ornament we are considering can be superposed with itself by rotating it around the axis of symmetry at an angle of 120°. Additionally, the ornament shown in Fig. 1, b, has translation symmetry with three translation axes defined by perpendiculars to the sides of a regular triangle. Consequently, it can be superposed with itself using translation in one of the three directions specified by the translation axes.

Let's show in Fig. 1, c is the third variant of the ornament, stylizing M. C. Escher’s lithograph ‘Reptiles’ and representing a figure that inscribes into a rhombus.

It is obvious that the ornament shown in Fig. 1, c does not have a single plane of symmetry, that is, it does not have a single reflection symmetry group. At the same time, it does not have a single axis of symmetry, that is, it does not have a single rotation symmetry group. This means that the ornament we are considering can be superposed with itself by rotating around the axis of symmetry only by an angle of 360°. At the same time, the ornament shown in Fig. 1, c, has translation symmetry with four translation axes defined by perpendiculars to the sides of the rhombus Consequently, it can be superposed with itself with translation in one of the four directions specified by the translation axes.



a b c

Fig. 1. Ornament, stylizing of M. C. Escher's lithograph 'Reptiles'

CONCLUSIONS

Thus, the article proposes a rule for constructing a figured tile that stylizes images of plants and animals and fills the plane without overlaps and gaps with translations and rotations of its repetitions, in particular a figured tile that generalizes images of a zoomorphic form in M. C. Escher's lithograph 'Reptiles'. The proposed rule was applied to compose an ornament stylizing M. C. Escher's lithograph 'Reptiles'. It is shown that this ornament has multitude axes of 3rd order symmetry and six translation axes. Additionally, a connection has been identified between the symmetry group of the ornament and the movements of the plane, leading to the formation of its figured tiles.

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НІЦИН О.

РОЗГАДКА ТАЄМНИЦІ ЛІТОГРАФІЇ М. К. ЕШЕРА «ЯЩІРКИ»

Запропоновано правило побудови фігурної плитки, що стилізує зображення рослин і тварин та заповнює площину без накладень та пропусків при паралельних переносах та обертаннях її повторень. Розглянуто побудову фігурної плитки, що узагальнює зооморфну форму на літографії М. К. Ешера «Ящірки». Запропоноване правило було застосовано для складання орнаменту, що стилізує літографію М. К. Ешера «Ящірки».

Ключові слова: *орнаменти, фігурні плитки у формі тварин та рослин, стилізація гравюр М. К. Ешера.*